GAUS AG ON THE DE RHAM–WITT COMPLEX
SUMMER SEMESTER 2026

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1. Time and place

Mondays, 10:15 to 11:45, in the Hilbert room.

2. References

The main reference is

- *Revisiting the de Rham–Witt complex* by B. Bhatt, J. Lurie and A. Mathew.
- *Cohomologie cristalline des schémas de caractéristique* $p > 0$ by P. Berthelot.

3. Program

If not mentioned otherwise, the numbering refers to the paper by BLM. “Discuss” means at least state and explain, and prove or give an idea of the proof if time permits; your talk should cover the section indicated in its title. The description on the right is there to guide you, and contains amongst other things results that are useful for the later talks.

13.4.	Introduction	Timo	Overview. Recollections on Witt vectors.
20.4. (14h15 –16h)	Recollections + §2: Dieudonné complexes 1/2	Tom	Define the de Rham complex (in general) and the completed de Rham complex (Variant 3.3.1 without the Dieudonné algebra structure). Discuss the Cartier (iso)morphism (Proposition 3.3.4 and Theorem 3.3.6). Define Dieudonné complexes and their saturation: 2.1.1, 2.1.3, 2.1.4, §2.2, §2.3.
27.4.	§2: Dieudonné complexes 2/2	Niklas	Some homological algebra. Define Dieudonné complexes of Cartier type (Definition 2.4.1) and discuss Theorem 2.4.2. Define completion of Dieudonné complexes (Construction 2.5.1) and strictness (Definition 2.5.4). Prove Proposition 2.6.5 in the particular case of $\mathcal{W}(M)^*$ for M^* saturated (without mentioning strict Dieudonné towers). Cover §2.7 and §2.8.
4.5.	§3: Dieudonné algebras	Andy	More homological algebra and our main examples. Define Dieudonné algebras (Definition 3.1.2). Discuss Examples 3.1.8, 3.2, and that of the completed de Rham complex as a Dieudonné algebra (Variant 3.3.1, Corollary 3.3.8). Discuss Propositions 3.4.2 and 3.4.3. Prove Propositions 3.5.5 and 3.5.8, and Corollaries 3.5.9 and 3.5.10.
11.5.	§4: Definition of the complex	Julie	The de Rham–Witt complex, its universal property, and relation to the de Rham complex. Cover §3.6 (discuss in particular Lemma 3.6.1 and Proposition 3.6.3). Define the saturated de Rham–Witt complex (Definition 4.1.1, Proposition 4.1.4). State Corollary 4.1.5. Prove Theorem 4.2.4. Prove Theorem 4.3.1 (make sure to state Corollary 4.3.5), and discuss Remark 4.3.6.
18.5.	§5: Extension to schemes	Nutsa	The de Rham–Witt complex as a sheaf. The main goal is to prove Theorem 5.2.2 and Proposition 5.2.4. In particular, cover §5.1. As time permits, discuss §5.3 (étale behaviour).

1.6.	§6: Seminormal aspects	Timon	The de Rham–Witt complex is invariant under semi-normalization (and recovers the semi-normalization of a ring). Cover §§6.1-6.2 and §§6.5-6.6, with a focus on Proposition 6.2.1, Corollary 6.5.2, and Theorem 6.5.3. As time permits, introduce crystalline cohomology (as the cohomology of the structure sheaf on the crystalline site relative to the divided power ring $(\mathbb{Z}_p, (p))$, see for example the book by Berthelot or Tag 07GI in the Stacks project) and cover §6.4.
8.6.	Derived considerations	Anton	The Nygaard filtration and foundations for the last four talks. Define filtered derived categories (Construction 8.4.1) and the Beilinson t-structure (Remark 8.4.8). Use Proposition 8.4.10 as a definition for the functor $L\eta_p$, and give a more explicit description relating it to the functor η_p of §2.1. Discuss Proposition 7.2.4, Theorem 7.4.7 and Corollary 7.4.8. Finally, define the Nygaard filtration as in Proposition 8.4.11. As time permits, come back to some descriptions and properties in §8.1 and §8.2.
15.6.	§9: The derived de Rham–Witt complex 1/2	Klaus	Alternative description of the de Rham–Witt complex. More general criterion for when it can recover the de Rham complex. Define the derived de Rham–Witt and derived de Rham complexes (Construction 9.2.5, Variant 9.2.6) and the conjugate filtration (Remark 9.2.7). Discuss Theorem 9.3.1 (the saturated de Rham–Witt complex in terms of the derived one). Prove Corollary 9.3.5 as an application. Prove Theorem 9.4.1 and Proposition 9.4.6.
22.6.	§9: The derived de Rham–Witt complex 2/2	Daniel	By the criterion of the previous talk, the de Rham–Witt complex recovers the de Rham complex for a more general class of rings. Cover §9.5, in particular Theorem 9.5.6, Corollary 9.5.19, and Theorem 9.5.21.
29.6.	§10: Comparison with crystalline cohomology 1/2	Lorenzo	The de Rham–Witt complex computes crystalline cohomology. If not done in the talk about §6, quickly introduce/recall crystalline cohomology (see for example the book by Berthelot or Tag 07GI in the Stacks project), state Theorem 10.1.1 and explain the strategy for the proof (§10.1). Prove Proposition 10.2.1; coordinate with the speaker of the next talk (they might need intermediate results and proofs from §10.2). If not done in the talk about §6 and time permits, prove Proposition 6.4.1 (which shows that the assumptions in the main result are needed).
6.7.	§10: Comparison with crystalline cohomology 2/2	Luca	End of the proof of the previous talk. Prove Propositions 10.3.1 and 10.4.1; show how everything fits together to prove Theorem 10.1.2. Coordinate with the speaker of the previous talk (you might need intermediate results and proofs from §10.2). If time permits and it was not done before, prove Proposition 6.4.1.